Lecture 9:
Radiometry and Cameras

Image Synthesis
Stanford CS348b, Spring 2014
Radiometry

- System of units and measures for measuring illumination
- Geometrical optics model of light
  - Photons travel in straight lines, represented by rays
  - Good approximation when wavelength $<<$ size of objects light interacts with
    - Conversely, doesn’t model diffraction, interference, ...
Visible electromagnetic radiation

- Light
- Visible electromagnetic radiation
Lights. How Do They Work?

- Physical process converts energy into photons
  - Each photon carries a small amount of energy
- Over some amount of time, light consumes some amount of energy, Joules
  - Some is turned into heat, some into light
- Energy ~ exposure
  - Film, sunburn, solar panels, ...
- Can also find instantaneous energy consumption, Watts (power, Joules/second)
Measuring Illumination: Flux

- Given a sensor, we can count how many photons pass through it
  - Over a period of time, gives the power received by the sensor
- Energy carried by a photon:
  \[ Q = \frac{hc}{\lambda} \]
  \[ h \approx 6.626 \times 10^{-34} \]
Measuring Illumination: Flux

- Given a sensor, we can count how many photons pass through it
  - Over a period of time, gives the power received by the sensor

- Taking the limit
  \[ \Phi = \lim_{\Delta \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad [\text{J/s}] \]
  gives flux (Watts) received by the sensor

- Conversely
  \[ Q = \int_{t_0}^{t_1} \Phi(t) \, dt \]
Measuring Illumination: Irradiance

- **Flux**: time density of energy
- **Irradiance**: area density of flux

Given a sensor of with some area, we can consider the average flux over the entire sensor area:

\[
\frac{\Phi}{A}
\]
Measuring Illumination: Irradiance

- Flux: time density of energy
- Irradiance: area density of flux

Given a sensor of with some area, we can consider the average flux over the entire sensor area:

\[
\frac{\Phi}{A}
\]

Irradiance is given by taking the limit of area at a single point on the sensor:

\[
E(p) = \lim_{\Delta \to 0} \frac{\Delta \Phi(p)}{\Delta A} = \frac{d\Phi(p)}{dA} \quad \left[ \frac{W}{m^2} \right]
\]
Beam Power In Terms of Irradiance

\[ \Phi = EA \]

\[ E = \frac{\Phi}{A} \]
Beam Power In Terms of Irradiance

\[ \Phi' = E' A' \]

\[ E' = \frac{\Phi'}{A'} \]
Projected Area

\[ A = A' \cos \theta \]
Lambert’s Cosine Law

\[ A = A' \cos \theta \]
\[ \Phi = \Phi' \]

\[ E' = \frac{\Phi'}{A'} = \frac{\Phi}{A} \cos \theta = E \cos \theta \]
Irradiance Falloff With Distance

Assume light is emitting flux $\Phi$ in a uniform angular distribution.

Compare irradiance at two spheres:

$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$
Irradiance Falloff With Distance

Assume light is emitting flux $\Phi$ in a uniform angular distribution

Compare irradiance at two spheres:

$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$
Irradiance Falloff With Distance

Assume light is emitting flux $\Phi$ in a uniform angular distribution.

Compare irradiance at two spheres:

$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2}$$
Angles and Solid Angles

- **Angle**: \( \theta = \frac{l}{r} \)
  - Circle has \( 2\pi \) radians

- **Solid angle**: \( \Omega = \frac{A}{r^2} \)
  - Sphere has \( 4\pi \) steradians
Solid Angles in Practice

- Earth and moon both subtend ~60µ sr as seen from earth
- Surface area of earth: ~510M km²
- Projected area:

\[
510 \text{Mkm}^2 \frac{60\mu\text{sr}}{4\pi \text{sr}} = 510 \frac{15}{\pi} \approx 2400 \text{km}^2
\]

http://xkcd.com/1276/
Solid Angles in Practice

http://xkcd.com/1276/
Differential Solid Angles
Differential Solid Angles

\[ r \sin \theta \]

\[ \theta \]

\[ \phi \]
Differential Solid Angles
Differential Solid Angles

\[ dA = (r \, d\theta) (r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) \]
\[ = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ d\omega = \sin \theta \, d\theta \, d\phi \]

\[ \Omega = \int_{S^2} d\omega \]

\[ = \int_0^\pi \int_0^{2\pi} \sin \theta \, d\theta \, d\phi \]

\[ = \int_{-1}^1 \int_0^{2\pi} d\cos \theta \, d\phi \]

\[ = 4\pi \]
Measuring Illumination: Radiance

- Radiance is the solid angle density of irradiance

\[ L(p, \omega) = \lim_{\Delta \to 0} \frac{\Delta E_\omega(p)}{\Delta \omega} = \frac{dE_\omega(p)}{d\omega} \left[ \frac{W}{m^2 \text{sr}} \right] \]

where \( E_\omega \) denotes that the differential area is oriented to face in the direction \( \omega \)
Radiance Functions

- Equivalently,

\[ L(p, \omega) = \frac{dE(p)}{d\omega \cos \theta} = \frac{d^2\Phi(p)}{dA \, d\omega \cos \theta} \]

- Have described in terms of measuring radiance at a surface
  - But \( dA \) is oriented to face the ray direction
  - \( \cos \theta \) accounts for possibly different surface orientation
Field Radiance: The Light Field

- Light field = radiance function on rays
- Radiance is constant along rays
- Spherical gantry: 4D light field

\[ L(x, y, \theta, \phi) \]

Capture all light leaving an object
Surface Radiance

- Need to distinguish between incident radiance and exitant radiance functions at a point on a surface.

\[ L_i(p, \omega) \]

\[ L_o(p, \omega) \]

\[ L(p, \omega) = ?? \]
Properties of Radiance

- Fundamental field quantity that characterizes the distribution of light in an environment
  - Radiance is the quantity associated with a ray
  - Rendering is all about computing radiance
- Radiance is invariant along a ray in a vacuum
- Response of a sensor is proportional to radiance
  - Cameras measure radiance
Irradiance From the Environment

\[ dE(p, \omega) = L_i(p, \omega) \cos \theta \, d\omega \]

\[ E(p, \omega) = \int_{H^2} L_i(p, \omega) \cos \theta \, d\omega \]
Irradiance from a Uniform Area Source

$$E(p) = \int_{H^2} L \cos \theta \, d\omega$$

$$= L \int_{\Omega} \cos \theta \, d\omega$$

$$= L \Omega^\perp$$

The projected solid angle is the number of rays leaving $A$ that intersect $dA$
Uniform Disk Source

Geometric Derivation

Algebraic Derivation

\[ \Omega^\perp = \pi \sin^2 \alpha \]

\[
\Omega^\perp = \int_1^{\cos \alpha} \int_0^{2\pi} \cos \theta \, d\phi \, d\cos \phi
\]

\[
= 2\pi \left[ \frac{\cos^2 \theta}{2} \right]_0^{\cos \alpha}
\]

\[
= \pi \sin^2 \alpha
\]

\[
= \pi \frac{r^2}{r^2 + h^2}
\]
Measuring Illumination: Radiant Intensity

- Power per solid angle emanating from a point source

\[ I(\omega) = \frac{d\Phi}{d\omega} \quad \left[ \frac{W}{\text{sr}} \right] \]
Isotropic Point Source

\[ \Phi = \int_{S^2} I \, d\omega \]
\[ = 4\pi I \]

\[ I = \frac{\Phi}{4\pi} \]
Goniometric Diagrams

http://www.visual-3d.com/tools/photometricviewer/
Rendering With Goniometric Diagrams

Luminaire 1 - Louis Poulsen 94-02511
File: IESlouis_poulsen.ies
File: IESlouis_poulsen.exr
Count: 342

Luminaire 2 - Siemens 5LJ180 7-1CD2 EVG
File: IESSiemens.ies
File: IESSiemens.exr
Count: 156

http://www.mpi-inf.mpg.de/resources/mpimodel/v1.0/luminaires/index.html
Rendering With Goniometric Diagrams

http://www.mpi-inf.mpg.de/resources/mpimodel/v1.0/gallery/cmpimages/index.html
Radiometric Relationships

\[ \Phi = \frac{dQ}{dt} \]

\[ E(p) = \frac{d\Phi(p)}{dA} \]

\[ L(p, \omega) = \frac{dE(p)}{d\omega \cos \theta} \]

\[ Q = \int \Phi(t) \, dt \]

\[ \Phi = \int E(p) \, dA \]

\[ E(p) = \int L(p, \omega) \cos \theta \, d\omega \]
Photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system
- Integrate over wavelengths, weight with luminous efficacy curve, e.g.:

\[ L_v(p, \omega) = 683.002 \int_0^\infty L(p, \omega, \lambda) \bar{y}(\lambda) \, d\lambda \]
<table>
<thead>
<tr>
<th>Physics</th>
<th>Radiometry</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Radiant Energy</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Flux (Power)</td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
<tr>
<td>Flux Density</td>
<td>Irradiance Radiosity</td>
<td>Illuminance Luminosity</td>
</tr>
<tr>
<td>Angular Flux Density</td>
<td>Radiance</td>
<td>Luminance</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radiant Intensity</td>
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# Photometric Units

<table>
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<tr>
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<td>Luminous Power</td>
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<tr>
<td>Illuminance Luminosity</td>
<td>Lux</td>
<td>Phot</td>
<td>Footcandle</td>
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<tr>
<td>Luminance</td>
<td>Nit, Apostlib, Blondel</td>
<td>Stilb Lambert</td>
<td>Footlambert</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela</td>
<td>Candela</td>
<td>Candela</td>
</tr>
</tbody>
</table>

“Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?” —James Kajiya
Cameras
Depth of Field

London and Upton,
Real Lens

Vivitar Series 1, 90mm f/2.5 lens
Cover photo of Kingslake, “Optics in Photography”
Thin Lens Model

Focal Length, $F$, is the distance behind the lens where parallel rays focus.

Lens diameter is $F/n$, where $n$ is the lens's f-number.
Gaussian Lens Formula

Points at distance $z$ focus at distance $z'$ behind the lens given by

\[
\frac{1}{F} = \frac{1}{z} + \frac{1}{z'}
\]

Gaussian Lens Formula

Points at distance $z$ focus at distance $z'$ behind the lens given by

$$\frac{1}{F} = \frac{1}{z} + \frac{1}{z'}$$

Lenses are focused by moving them closer to/farther from the image plane.

Defocus Blur

Lens is focused at depth $z_f$; points at other depths image to an area on the image plane: the circle of confusion
Defocus Blur

Lens is focused at depth $z_f$; points at other depths image to an area on the image plane: the circle of confusion
Finding the Size of the Circle of Confusion

What is \( d_c \), the diameter of the circle of confusion of a point at depth \( z \)?

We know \( z_i' \), the image plane position, and the lens diameter \( d \)

We can compute \( z' \) from \( F \) and \( z \) \[
\frac{1}{F} = \frac{1}{z} + \frac{1}{z'}
\]

Similar triangles: \[
d_c/(z_i'-z') = d/z'
\]

Solve: \[
d_c = d | z_i' - z' | (1/F - 1/z)
\]
Finding the Size of the Circle of Confusion

What is $d_c$, the diameter of the circle of confusion of a point at depth $z$?

We know $z'_i$, the image plane position, and the lens diameter $d$.

We can compute $z'$ from $F$ and $z$:

$$\frac{1}{F} = \frac{1}{z} + \frac{1}{z'}$$

Similar triangles:

$$\frac{d_c}{(z'_i-z')} = \frac{d}{z'}$$

Solve:

$$d_c = d | z'_i - z' | (1/F - 1/z)$$
Circle of Confusion as a function of $z$

$$d_c = \frac{d|z_i' - z'|}{1/F - 1/z}$$
Ray Tracing for Defocus Blur (Thin Lens)

- Rays pass from film plane through points on the lens
- All rays pass through the plane of focus at $z_f$
  - This fact determines the actual ray directions
- Ray traced defocus blur doesn’t have any more expensive per-sample test (different than rasterizing in this respect)
Ray Tracing for Defocus Blur (Thin Lens)

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**Double Gauss Lens**

<table>
<thead>
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<th>Radius (mm)</th>
<th>Thick (mm)</th>
<th>n</th>
<th>V-no</th>
<th>aperture</th>
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From W. Smith, *Modern Lens Design*, p. 312
Ray Tracing Through Lenses

200 mm telephoto

35 mm wide-angle

50 mm double-gauss

16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)
Film and Sensors

- Camera sensors (film, CCD, ...) effectively count how many photons hit them over small areas of the image.
- Want to determine how much energy arrives over regions of the sensor over a given period of time:
  - Integrate irradiance over area to get power (flux)
  - Integrate flux over time to get energy (joules)

\[ Q = \int_{t_0}^{t_1} \int_A E(p', t') \, dp' \, dt' \]

- How do we compute irradiance at a point on the image plane?
The Measurement Equation

Lens Aperture

Film Plane

\[ E(p, t) = \int_{H^2} L_i(p, \omega', t) \cos \theta \, d\omega' \]
The Measurement Equation

Lens Aperture

Film Plane

\[ E(p, t) = \int_A L(p' \rightarrow p, t) \frac{\cos \theta \cos \theta'}{||p' - p||^2} \, dA' \]
The Measurement Equation

Lens Aperture

Film Plane

\[ E(p, t) = \int_A L(p' \rightarrow p, t) \frac{\cos \theta \cos \theta'}{||p' - p||^2} \, dA' \]

\[ = \int_A L(p' \rightarrow p, t) \frac{\cos^2 \theta}{||p' - p||^2} \, dA' \]

Assume aperture and film plane are parallel.
The Measurement Equation

Lens Aperture

\[ ||p' - p|| = \frac{d}{\cos \theta} \]

Film Plane

\[ E(p, t) = \int_A L(p' \to p, t) \frac{\cos^2 \theta}{||p' - p||^2} \, dA' \]

\[ = \frac{1}{d^2} \int_A L(p' \to p, t) \cos^4 \theta \, dA' \]
The Measurement Equation

Lens Aperture

\[ ||p' - p|| = \frac{d}{\cos \theta} \]

Film Plane

\[ Q = \frac{1}{d^2} \int_{t_0}^{t_1} \int_{A_{\text{lens}}} \int_{A_{\text{film}}} L(p' \rightarrow p, t) \cos^4 \theta \, dp \, dp' \, dt' \]
Motion Blur

Cook et al. 1984

Hou et al. 2010
Ray Traced Motion Blur

- Each ray from the camera has an associated time
- Conceptually: before intersecting with scene, transform all moving objects to the ray’s time
  - Intersect ray, do lighting calculations, etc. at that time
Evaluating the Measurement Equation

\[ Q = \frac{1}{d^2} \int_{t_0}^{t_1} \int_{\Omega_{\text{lens}}} \int_{A_{\text{film}}} L_i(p', \omega', t') \cos^4 \theta' \, dp' \, d\omega' \, dt' \]

The incident radiance at the film plane is equal to the exitant radiance at first visible surface (ignoring absorption in lenses)

\[ L_i(p, \omega_i) \]

\[ \omega_o = -\omega_i \]

\[ L_o(p, \omega_o) \]
Evaluating the Measurement Equation

\[ Q = \frac{1}{d^2} \int_{t_0}^{t_1} \int_{\Omega_{\text{lens}}} \int_{A_{\text{film}}} L_i(p', \omega', t') \cos^4 \theta' \, dp' \, d\omega' \, dt' \]

\[ \approx \frac{(t_1 - t_0) A_{\text{film}} A_{\text{lens}}}{d^2} \sum_{i}^{N_i} \sum_{j}^{N_j} \sum_{k}^{N_k} L_i(p_i, \omega_j, t_k) \cos^4 \theta_j \]

Problem: the “curse of dimensionality”
(Stay tuned!)
Light Field Photography

- Traditional photography: each pixel sensor computes ~average color seen over lens area
- Instead, preserve angular light distribution across different pixels

Light Field Photography