Lecture 13:
Surface Reflection I

Image Synthesis
Stanford CS348b, Spring 2013
Reflection Models

Definition: reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency

Properties:
- Spectra and color
- Polarization
- Directional distribution
Reflection Models

- Approaches:
  - Phenomenological
  - Physical
  - Measurement
  - Simulation
Types of Reflection Functions

- Ideal specular
  - Reflection law
  - Mirror
- Ideal diffuse
  - Lambert’s law
  - Matte
- Specular
  - Glossy
  - Directional Diffuse
The BRDF

- **Bidirectional Reflectance Distribution Function**

\[
f_r(\omega_i \to \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{dL_i(\omega_i) \cos \theta_i} \left[ \frac{1}{\text{sr}} \right]
\]
The Reflection Equation

\[ dL_r(\omega_r) = f_r(\omega_i \rightarrow \omega_r) \, dL_i(\omega_i) \, \cos \theta_i \]

\[ L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) \, L_i(p, \omega_i) \, \cos \theta_i \, d\omega_i \]
Solving The Reflection Equation

\[ L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i \, d\omega_i \]

- Basic Monte Carlo estimate:
  - Generate directions \( \omega_j \) sampled from some distribution \( p(\omega) \)
  - Match BRDF, or incident raduance function
  - Compute estimator

\[
\frac{1}{N} \sum_{j=1}^{N} \frac{f_r(p, \omega_j \rightarrow \omega_r) L_i(p, \omega_j) \cos \theta_j}{p(\omega_j)}
\]

- (We’ll go into depth on this in 2 lectures...)
Properties of BRDFs

- Linearity

[Sillion et al. 1990]

- Reciprocity principle

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]
Properties of BRDFs

- Isotropic vs. anisotropic
  - If isotropic, \( f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i) \)
  - Then, from reciprocity,
    \[
    f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)
    \]

- Energy conservation
Reflectance and Energy Conservation

\[ f_r(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{\text{sr}} \right] \]

\[
\frac{d\Phi_r}{d\Phi_o} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r \, d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i \, d\omega_i} \\
= \int_{\Omega_r} \int_{\Omega_i} \frac{f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r}{\int_{\Omega_i} \cos \theta_i \, d\omega_i} \\
\leq 1 \\
\int_{H^2} f_r(\omega_i \rightarrow \omega_r) \cos \theta_i \, d\omega_i \leq 1 \]
Perfect Specular Reflection

\[ \omega_o + \omega_i = 2 \cos \theta \vec{n} = 2(\omega_i \cdot \vec{n})\vec{n} \]

\[ \omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n} \]

\[ \phi_o = (\phi_i + \pi) \mod 2\pi \]
Specular Reflection BRDF

\[ L_i(\theta_i, \phi_i) \quad L_o(\theta_o, \phi_o) \]

\[ \theta_i \quad \theta_o \]

\[ L_o(\theta_o, \phi_o) = L_i(\theta_o, \phi_o \pm \pi) \]

\[ f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta \left( \cos \theta_i - \cos \theta_o \right)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) \]
Specular Reflection & The Reflection Equation

\[
L_o(\theta_o, \phi_o) = \int f_r(\theta_i, \phi_i; \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos \theta_i \, d \cos \theta_i \, d\phi_i \\
= \int \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) L_i(\theta_i, \phi_i) \cos \theta_i \, d \cos \theta_i \, d\phi_i \\
= L_i(\theta_r, \phi_r \pm \pi)
\]

Whitted’s method!
Snell’s Law

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

\[ \eta_i (\vec{n} \times \omega_i) = \eta_t (\vec{n} \times \omega_t) \]
Law of Refraction

\[ \omega_t = \mu \omega_i + \gamma \vec{n} \]

\[ |\omega_t|^2 = 1 = \mu^2 = \gamma^2 + 2\mu\gamma(\omega_i \cdot \vec{n}) \]

\[ \gamma = -\mu(\omega_i \cdot \vec{n}) \pm [-\mu^2(1 - (\omega_i \cdot \vec{n})^2)]^{\frac{1}{2}} \]

\[ = \mu \cos \theta_i \pm [1 - \mu^2 \sin^2 \theta_i]^{\frac{1}{2}} \]

\[ = \mu \cos \theta_i \pm \cos \theta_t \quad \leftarrow \gamma = \mu - 1 \]

\[ = \mu \cos \theta_i - \cos \theta_t \]

Total Internal Reflection:

\[ 1 - \mu^2(1 - (\omega_i \cdot \vec{n})^2) < 0 \]
Optical Manhole

Total Internal Reflection

[Livingston and Lynch]
Fresnel Reflection

- Reflections from a shiny floor

[Lafortune et al. 1997]
Fresnel Reflection (Dielectric, \(\eta = 1.5\))
Fresnel Reflectance (Conductor)
Glass with Fresnel Reflection/Transmission
Without Fresnel
(bug)
Lambertian Reflection

- Assume light is equally likely to be reflected in each output direction

\[ L_o(\omega_o) = \int_{H^2} f_r L_i(\omega_i) \cos \theta_i \, d\omega_i \]

\[ = f_r \int_{H^2} L_i(\omega_i) \cos \theta_i \, d\omega_i \]

\[ = f_r E \]

\[ f_r = \frac{\rho}{\pi} \]
Hemispherical Incident Radiance
Hemispherical Incident Radiance
Hemispherical Incident Radiance

- At any point on any surface in the scene, there's an incident radiance field that gives the directional distribution of illumination at the point.

\[ L_i(\omega) = L_i(x, y, \sqrt{1 - x^2 - y^2}) \]
Specular Reflection

Incident Radiance

Exitant Radiance
Diffuse Reflection

Incident Radiance

Exitant Radiance
Plastic

Incident Radiance

Exitant Radiance
Copper

Incident Radiance

Exitant Radiance
Measuring BRDFs: Motivation

- Avoid need to develop / derive models
  - Automatically includes all of the scattering effects present
- Can accurately render with real-world materials
  - Useful for product design, special effects, ...
- Theory vs. practice:

[Bagher et al. 2012]
Measuring BRDFs

- General approach:

  ```
  foreach outgoing direction wo
      move light to illuminate surface with a thin beam from wo
    for each incoming direction wi
      move sensor to be at direction wi from surface
      measure incident radiance
  ```

- Improving efficiency:
  - Isotropic surfaces reduce dimensionality from 4D to 3D
  - Reciprocity reduces # of measurements by half
  - Clever optical systems..
Measuring BRDFs: Gonioreflectometer

[Li et al. 2005]
Measuring BRDFs: Mirrored Hemisphere

[Ward 1994]
Measuring BRDFs

[Ben-Ezra et al. 2008]
Image-Based BRDF Measurement

[Marschner et al. 1999]
Challenges in Measuring BRDFs

- Accurate measurements at grazing angles
  - Important due to Fresnel effects
- Measuring at a high-enough rate to capture specularities
- Retro-reflection
- Oxidization, spatially-varying reflectance, ...
Representing Measured BRDFs

Desirable qualities

- Compact representation
- Accurate representation of measured data
- Efficient evaluation for arbitrary pairs of directions
- Good distributions available for importance sampling
Tabular Representation

- Store regularly-spaced samples in \((\theta_i, \theta_o, |\phi_i - \phi_o|)\)
  - Better: reparameterize angles to better match specularities
- Generally need to resample measured values to table
- Very high storage requirements

MERL BRDF Database
[Matusik et al. 2004]
90*90*180 measurements
Basis Functions

- Can fit existing models, e.g. Cook-Torrance, 3 parameters per wavelength, $k_d, k_s, k_e$

$$f_r(\omega_i \rightarrow \omega_o) = k_d + k_s (\vec{h} \cdot \vec{n})^{k_e} \quad \vec{h} = \omega_i + \omega_o$$

- More sophisticated, e.g. [Bagher et al. 2012] 11 parameter model:
Simulation: Velvet

[Westin et al. 1992]
Simulation: Brushed Aluminum

[Westin et al. 1992]
Simulation: Nylon

[Westin et al. 1992]
Simulation: Paint

[Gondek et al. 1994]